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Detecting multiple breaks in time series covariance structure: a non-parametric approach based on the evolutionary spectral density

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This article estimates the number of breaks and their locations in the covariance structure of a series based on the evolutionary spectral density and uses some standard information criteria. The adopted approach is non-parametric and does not privilege a priori any modelling of the series. One carries out a Monte Carlo analysis and an empirical illustration using the daily return series of exchange rate euro/US dollar to support the relevance of the theory and to produce additional insights. The simulation results are globally adequate and show that the criteria having heavy penalty are more accurate in the selection of the number of breaks. The empirical results indicate that the covariance structure of the return series considerably varies between 30 March 2000 and 6 April 2001. The unconditional volatility appears non-constant over this interval.

I. INTRODUCTION

The question of selecting the number of changes in the level (mean-shifts) or trend of a time series acquires a capital importance in the literature. Indeed, Yao (1988), Yao and Au (1989) and Yin (1988) have considered models with structural change in the level and have estimated the number of mean-shifts using the Bayesian information criterion. Ben Aïssa and Jouini (2003) evoked the instability problem when the change affects the level and the persistence of an autoregressive process of order 1 and used some standard information criteria to estimate the number of breaks and their locations in the US inflation. They found economic explanations to show why in the detected dates there are changes in the US inflation process and their results show in particular that the evolution curve of the inflation was flattened during the last 20 years since it is noted that this reduction in extent of inflation is stable and durable.

All this literature focuses on the instability problems in the time of the first moment of the series. However, this article is interested in detecting regime shifts of the second moment, that is the covariance function based on the evolutionary spectral density. The practical importance of instability of the second moment has been highlighted through an assessment of several economic and financial series. In this context, Pagan and Schwert (1990) studied the instability of the covariance structure of monthly US stock return series. Using series of large size, Mikosch and Starica (2004) explained some classic stylized facts (long-range dependence) by regime shifts of the unconditional volatility.

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The purpose of this article is to estimate the number and the locations of breaks corresponding to regime shifts in the covariance structure of the studied series. It focuses on series having different stationarity forms in several successive intervals. Because the covariance function of stationary process is the Fourier transform of the spectral density, one of the best approaches to studying the stability of the covariance structure is to studying the stability of the spectral density. This approach has been adopted by von Sachs and Neumann (2000), and then by Ahamada and Boutahar (2002) to develop stationarity tests of the covariance structure. This article follows such work. First, it estimates a time spectral density based on the theory of the evolutionary spectrum of Priestley (1965, 1996). Then, it estimates the number of breaks and their locations by examining changes in spectral density form.

The article is organized as follows: Section II recalls the theory of the evolutionary spectrum of Priestley (1965). Section III shows how the evolutionary spectral density can be used to locate the structure changes of the covariance function. It also surveys the structural change model and some selection procedures allowing to estimate the number of breaks. The heart of the article is Sections IV and V where some Monte Carlo experiments and an empirical application using the daily return series of exchange rate euro/US dollar are performed. The simulation results are globally adequate and the empirical results reveal some structural changes in the unconditional volatility of the return series of exchange rate. These last results show again that the treatment of long financial series using classic tools implying the second moment stationarity (long memory, family of models ARCH) is not always justified and in this case models with variable coefficients can be adapted. It is in this direction that these results are brought closer to those obtained by Mikosch and Starica (2004) or by Loretan and Phillips (1994). Some concluding comments are offered in Section VI.

II. THEORY OF THE EVOLUTIONARY SPECTRUM

Definition

Priestley's (1965) theory of the evolutionary spectrum is concerned with oscillatory processes, that is processes, $\{X_t\}$ defined as follows:

$$X_t = \int_{-\pi}^{\pi} A_t(\omega) e^{i\omega t} dZ(\omega) \quad (1)$$

where for each ω, the sequence $\{A_t(\omega)\}$, as function of t, has a generalized Fourier transform whose modulus has an absolute maximum at the origin. $\{Z(\omega)\}$ is an orthogonal process on $[-\pi, \pi]$ with $E[dZ(\omega)] = 0$,[1] $E[|dZ(\omega)|^2] = d\mu(\omega)$, where $\mu(\omega)$ is a positive measure. Without loss of generality, the evolutionary spectral density of the process $\{X_t\}$ is given by:

$$h_t(\omega) = \frac{dH_t(\omega)}{d\omega}, \quad -\pi \le \omega \le \pi \quad (2)$$

where $dH_t(\omega) = |A_t(\omega)|^2 d\mu(\omega)$. Priestley's evolutionary spectrum theory is a particularly attractive concept since it has a physical interpretation. It encompasses most other approaches as special cases and includes many types of non-stationary processes. The instantaneous variance of $\{X_t\}$ is given by:

$$\sigma_t^2 = \mathrm{var}(X_t) = \int_{-\pi}^{\pi} h_t(\omega)\, d\omega \quad (3)$$

These relations show that all modifications in the time of the covariance structure of the studied series may be captured by studying the stability of the evolutionary spectral density $h_t(\omega)$. In particular, the relation given by Equation 3 shows that a modification of the variance of the process necessarily entails a variation of $h_t(\omega)$ with respect to the variable time.

Estimation of the evolutionary spectral density

An estimator of $h_t(\omega)$ at time t and frequency ω can be obtained using two windows $\{g_u\}$ and $\{\omega_\upsilon\}$. Without loss of generality, the estimator $\hat{h}_t(\omega)$ is constructed as follows:

$$\hat{h}_t(\omega) = \sum_{\upsilon \in Z} \omega_\upsilon |U_{t-\upsilon}(\omega)|^2 \quad (4)$$

where $U_t(\omega) = \sum_{u \in Z} g_u X_{t-u} e^{-i\omega(t-u)}$. One chooses the following windows $\{g_u\}$ and $\{w_v\}$:

$$g_u = \begin{cases} 1/(2\sqrt{h\pi}), & \text{if } |u| \le h, \\ 0, & \text{if } |u| > h, \end{cases} \quad \text{and } \omega_v = \begin{cases} 1/T', & \text{if } |\upsilon| \le T'/2 \\ 0, & \text{if } |\upsilon| > T'/2 \end{cases} \quad (5)$$

From Priestley (1988), $E(\hat{h}_t(\omega)) \simeq h_t(\omega)$, $\mathrm{var}(\hat{h}_t(\omega))$ decreases as T' increases and $\forall(t_1, t_2)$, $\forall(\omega_1, \omega_2)$, $\mathrm{cov}[\hat{h}_{t_1}(\omega_1)\hat{h}_{t_2}(\omega_2)] \approx 0$ if at least one of the following conditions (*i*) or (*ii*) is satisfied:[2]

$$\text{(i) } |t_1 - t_2| \ge T' \quad \text{(ii) } |\omega_1 \pm \omega_2)| \ge \frac{\pi}{h} \quad (6)$$

[1] This condition implies that $E(X_t) = 0$.

[2] For more details on the relations (i) and (ii) and the choice of h and T', the readers are referred to Priestley and Rao (1969).

III. DETECTION OF BREAKS IN THE COVARIANCE STRUCTURE

Let $\{X_t\}_{t=1}^{T}$ be data from a discrete process $\{X_t\}$ with theoretical evolutionary spectral density $h_t(\omega)$. One considers the grid of times $\{t_i = T'i\}_{i=1}^{I}$, where $I=[T/T']$ ([.]) denotes the integer part of argument) and the grid of frequencies $\{\omega_j = (\pi/20)(1+3(j-1))\}_{j=1}^{h}$. This implies that $\{t_i\}$ and $\{\omega_j\}$ satisfy the above-mentioned conditions (i) and (ii). Let $Y_{ij} = \ln(\hat{h}_{t_i}(\omega_j))$, and $h_{ij} = \ln(h_{t_i}(\omega_j))$. From Priestley and Rao (1969), one has:

$$Y_{ij} \approx h_{ij} + e_{ij} \tag{7}$$

where the sequence $\{e_{ij}\}$ is approximately uncorrelated and identically distributed normal. Equation 7 may also be written as:

$$Y_{i.} \approx h_{i.} + e_{i.} \tag{8}$$

where $Y_{i.} = (1/h)\sum_{j=1}^{h} Y_{ij}$ and $h_{i.} = (1/h)\sum_{j=1}^{h} h_{ij}$. In other words, $Y_{i.}$ and $h_{i.}$ are respectively the means of logarithm of the estimated and theoretical spectral densities They are means on the frequency grid $\{\omega_j = (\pi/20)(1+3(j-1))\}_{j=1}^{h}$. The number of taken values $h_{i.}$ depends on the number of regime shifts of the series $\{X_t\}_{t=1}^{T}$. Indeed, on each interval where the series is stationary, the evolutionary spectral density is independent of time, that is $h_{i.}$ is constant with respect to i on each of the intervals corresponding to regime-shifts. The model is then with change in mean (mean-shift model). More precisely, if the series is stationary on a sequence of $m+1$ successive intervals $\{I_l\}_{l=1}^{m+1}$, with $I_l \subset \{t_i = T'i\}_{i=1}^{I}$, $I_l \cap I_{l'} = \emptyset$ if $l \neq l'$ and $U_{l=1}^{m+1}\{I_l\} = \{t_i = T'i\}_{i=1}^{I}$, then the model given by Equation 8 becomes a mean-shift model with m breaks for the value of the spectral density $h_{i.}$:

$$Y_{i.} \approx h_l + e_{i.} \tag{9}$$

where h_l is a constant,[3] $h_l = h_{i.}$ for all $t_i \in I_l$. One applies some selection procedures to this model so as to estimate the number of structural breaks and their locations. Note that the dates corresponding to structural changes will be estimated among the points of the grid $\{t_i = T'i\}_{i=1}^{I}$. In the remainder of this section we describe the structural change model, the estimation method allowing to estimate the unknown parameters, and the selection procedures.

Structural change model and estimators

Consider the following structural change model with m mean-shifts:

$$Y_{i.} = h_l + e_{i.}, \quad i \in I_l \tag{10}$$

where $I_l = \{(k_{l-1}+1)T', \ldots, k_l T'\}$ for $l=1,\ldots,m+1$ with $k_0=0$ and $k_{m+1}=I$. $Y_{i.}$ is the observed dependent variable, h_l $(1 \leq l \leq m+1)$ are the regression coefficients with $h_\iota \neq h_{\iota+1}$ $(1 \leq \iota \leq m)$; and $e_{i.}$ is the disturbance. The break dates $(k_1, \ldots, k_m)$ are explicitly treated as unknown. Note that this is a pure structural change model where all the coefficients are subject to change. The purpose is to estimate the unknown regression coefficients and the break dates $(h_1, \ldots, h_{m+1}, k_1, \ldots, k_m)$ when I observations on $Y_{i.}$ are available. Let $h = (h_1, h_2, \ldots, h_{m+1})'$.

The estimation method considered is that based on the least-squares principle proposed in Bai and Perron (1998). This method is described as follows. For each m-partition $(k_1, \ldots, k_m)$, denoted $\{k_l\}$, the associated least-squares estimate of h_l is obtained by minimizing the sum of squared residuals $\sum_{l=1}^{m+1}\sum_{i=k_{l-1}+1}^{k_l}(Y_{i.} - h_l)^2$. Let $\hat{h}(\{k_l\})$ denote the resulting estimate. Substituting it in the objective function and denoting the resulting sum of squared residuals as $S_I(k_1, \ldots, k_m)$, the estimated break dates $(\hat{k}_1, \ldots, \hat{k}_m)$ are obtained as:

$$(\hat{k}_1, \ldots, \hat{k}_m) = \arg \min_{(k_1,\ldots,k_m)} S_I(k_1, \ldots, k_m)$$

where the minimization is taken over all partitions $(k_1, \ldots, k_m)$ such that $k_\iota - k_{\iota-1} \geq \theta$.[4] Thus the break date estimators are global minimizers of the objective function. Finally, the estimated regression parameters are the associated least-squares estimates at the estimated m-partition $\{\hat{k}_l\}$, that is $\hat{h} = \hat{h}(\{\hat{k}_l\})$. For our Monte Carlo exploration and empirical illustration, we use the efficient algorithm developed in Bai and Perron (2003) based on the principle of dynamic programming which allows global minimizers to be obtained using a number of sums of squared residuals that is of order $O(I^2)$ for any $m \geq 2$.

Model selection procedures

A common procedure for the selection of a model dimension is to consider an information criterion. Schwarz (1978) proposes the following criterion:

$$SIC(m) = \ln(S_I(\hat{k}_1, \ldots, \hat{k}_m)/(I-m)) + 2p^* \ln(I)/I$$

where $p^* = 2m+1$ is the number of unknown parameters. Yao (1988) uses the Bayesian information criterion defined as:

$$BIC(m) = \ln(S_I(\hat{k}_1, \ldots, \hat{k}_m)/I) + p^* \ln(I)/I$$

He showed that the estimator of the number of changes $\hat{m}$ is consistent (at least for normal sequence of random variables with shifts in mean) for m^0, the true number of breaks, provided $m^0 \leq M$ with M a known upper bound

[3] This is because the spectral density is independent of time in each interval where the process is stationary.
[4] Note that θ is the minimal number of observations in each segment ($\theta \geq 1$, not depending on I). From Bai and Perron (2003), if tests are required, then θ must be set of $[\varepsilon I]$ for some arbitrary small positive number ε.

for m. Another criterion proposed by Yao and Au (1989) is given by:

$$YIC(m) = \ln(S_I(\hat{k}_1, \ldots, \hat{k}_m)/I) = mC_I/I$$

where $\{C_I\}$ is any sequence satisfying $C_I I^{-2/n} \to \infty$ and $C_I/I \to 0$ as $I \to \infty$ for some positive integer n. The error term is with finite $2n$th moment for any $n \geq 3$. In our simulation experiments, we use the sequence $C_I = 0.368I^{0.7}$ proposed by Liu *et al.* (1997) who suggest a modified Schwarz' criterion that takes the form:

$$MIC(m) = \ln(S_I(\hat{k}_1, \ldots, \hat{k}_m)/(I - p^*)) + p^* c_0[\ln(I)]^{2+\delta_0}/I$$

They suggest using $c_0 = 0.299$ and $\delta_0 = 0.1$ based on the performance of the estimator of the number of breaks obtained by the criterion MIC for various simulation experiments carried out with several models. Note that these information criteria cannot directly take into account the effect of different distributions of the data and the errors across subsamples and possible serial correlation in the disturbances. The estimated number of break dates $\hat{m}$ is determined by minimizing the above-mentioned information criteria given M a fixed upper bound for m.

Nunes *et al.* (1996) showed that the criterion BIC tends to select the maximum possible number of changes for an integrated process of order 1 without breaks when one estimates a model with change in mean and change in trend. Perron (1997) studied, using simulations, the behaviour of the information criteria BIC and MIC in the context of estimating the number of breaks in the trend function of a series in the presence of serial correlation. These criteria perform reasonably well when the errors are uncorrelated but choose a number of changes much higher than the true value when serial correlation is present. When the errors are uncorrelated but a lagged dependent variable is present, the criterion BIC performs badly when the coefficient on the lagged dependent variable is large (and more so as it approaches unity). In such cases, the criterion MIC performs better under the null hypothesis of no break but underestimates the number of structural breaks when some are present. His results show that the conclusions of Nunes *et al.* (1996) do not depend on the fact that the data-generating process is a random walk; even an AR(1) process with a correlation degree smaller than one leads to an overestimation of the number of breaks. In the same context, Boutahar and Jouini (2003) provided mathematical proof and simulation evidence that for a trend-stationary process and a stationary AR(p) process without any structural break, the information criteria outlined above spuriously lead to the estimation of a number of changes higher than the true value. They also found that this bias towards the overestimation is less severe for the criteria having a heavy penalty. There are other works in the literature which use these criteria to determine the number of breaks and their locations based on economic and financial data. Among these works we find Bai and Perron (2003) and Jouini and Boutahar (2003).

All the cited works treat the instability problem in the time of the first moment of the series. However, the simulations and the empirical assessment carried out in this article are designed to the selection of the number and the locations of the breaks in the covariance structure of the series using the above-mentioned information criteria.

IV. MONTE CARLO EXPLORATION

This section reports some Monte Carlo experiments to implement the theoretical results outlined above. It considers two data-generating processes (DGPs) allowing for respectively the presence of one and two breaks in the variance. To compute the estimator of the evolutionary spectral density, the parameters of the windows $\{g_u\}$ and $\{w_v\}$ are set at $h = 7$,[5] and $T' = 20$. The minimal number of observations in each segment θ and the maximum permitted number of breaks M take value 5. The number of Monte Carlo replications is set at $N = 200$, a smaller value than one would recommend using in practice because Monte Carlo experiments in this case are more time consuming. Then long computing time presents a potentially serious barrier to study the behaviour of the information criteria in selecting the number of regime-shifts in the covariance structure of a series based on the evolutionary spectral density.

Case of one break

In a first time one supposes that the true DGP for X_t contains one break in the variance:

$$X_t = \begin{cases} N(0,1), & \text{if} \quad 1 \leq t \leq T_1^0 \\ N(0,4), & \text{if} \quad T_1^0 < t \leq T \end{cases} \tag{11}$$

where $N(\mu, \sigma^2)$ is the normal distribution with mean μ and variance σ^2, T_1^0 is the true break date and takes value $T/2$. The sample size is set at $T = 800$ and then the time grid on which we attempt to detect break points will be $\{t_i = 20i\}_{i=1}^{I}$, where $I = 800/20 = 40$. The estimation of the model given by Equation 10 provides the results presented in Table 1.

[5] The same selection as Artis *et al.* (1992) was adopted.

Table 1. *Percentage of breaks selected by the information criteria*

$\hat{m}$	SIC	BIC	YIC	MIC
0	0.0	0.0	0.0	0.0
1	93.5	58.5	36.5	84.5
2	6.0	26.0	36.5	14.0
3	0.5	15.0	25.5	1.5
4	0.0	0.5	1.5	0.0
5	0.0	0.0	0.0	0.0

Table 2. *Percentage of breaks selected by the information criteria*

$\hat{m}$	SIC	BIC	YIC	MIC
0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0
2	90.5	54.0	39.0	80.5
3	8.5	29.0	30.5	16.0
4	1.0	15.5	27.0	3.0
5	0.0	1.5	3.5	0.5

Case of two breaks

One now looks at the simulation results where the DGP contains two structural breaks in the variance:

$$X_t = \begin{cases} N(0,1), & \text{if} \quad 1 \le t \le T_1^0 \\ N(0,9/4), & \text{if} \quad T_1^0 < t \le T_2^0 \\ N(0,1), & \text{if} \quad T_2^0 < t \le T \end{cases} \tag{12}$$

where $T_1^0 = T/3$ and $T_2^0 = 2T/3$. Here the sample size is fixed at $T = 1200$ and then the time grid on which we try to select break dates will be $\{t_i = 20i\}_{i=I}^{I}$, where $I = 1200/20 = 60$. The results corresponding to the estimation of the model given by Equation 10 are reported in Table 2.

For the two models, the criteria BIC and YIC are biased and the estimators of the number of break dates obtained by these criteria have some distribution on the set $\{m^0, m^0+1, \ldots, M\}$ where the frequency of selecting the true number m^0 is the highest but remains even so low. On the other hand, the criteria SIC and MIC are more accurate and perform reasonably well since they select the true number of changes m^0 with large proportions. Note that the simulation results also show that the break date estimators are consistent since they converge to their true values.[6] Globally, the obtained results are satisfactory.

V. EMPIRICAL ILLUSTRATION

Ben Aïssa *et al.* (2004) used a test similar to the one based on the Kolmogorov–Smirnov statistic applied to the evolutionary spectrum, and Bai and Perron's (1998) selection procedure based on a sequence of tests to estimate the number of breaks and their locations in the monthly US inflation series covering the period 1957:1–2003:4. The obtained results of the two approaches are similar and economically significant. They also find that inflation was perfectly stable and durable during the 1990s. However, at the beginning of the 2000s the US economy was marked by a light recession expressed by a decrease of productivity and an increase in unemployment and inflation.

One now applies the above-mentioned approaches to the return series of exchange rate euro/US dollar. One considers daily data from 5 January 1999 to 20 February 2003 (yielding 1056 observations) obtained from the St Louis Reserve Federal Bank database. As for the simulation study, $h = 7$, $T' = 20$, and θ and M take value 5. The information criteria BIC, YIC and MIC select two breaks located in 30 March 2000 and 6 April 2001 while the criterion SIC detects zero change. Note that the 95% confidence intervals for the two breaks are respectively (10 November 1999–27 June 2000) and (2 May 2000–1 August 2001).[7] One remarks that the first break date is more precisely estimated than the second date since his 95% confidence interval covers a smaller period. These reported confidence intervals allow for different distributions of both the regressors and the errors in the different segments and the possibility of serial correlation in the disturbances. The heteroscedasticity and autocorrelation consistent covariance matrix is constructed following Andrews (1991) using a quadratic kernel with automatic bandwidth selection based on an AR(1) approximation. We also allow using pre-whitening as suggested in Andrews and Monahan (1992).[8]

These results show that the covariance structure of the return series of exchange rate euro/US dollar considerably varies between 30 March 2000 and 6 April 2001. The unconditional volatility of the series appears no constant on the considered interval. Indeed, the empirical variances on the three segments are respectively 3.9×10^{-5}, 7.7×10^{-5} and 3.8×10^{-5}. One remarks that the ones of the first and the last segments are almost identical and a look at the graph of the series (Fig. 1) might confirm this. Another feature of substantial importance and that might produce an additional support to these observations is that the estimated values of h_l $(1 \le l \le 3)$ are the same on the

[6] The corresponding results are not reported here and are available upon request from the authors.
[7] These confidence intervals are computed using the asymptotic distribution of the estimated break dates derived in Bai and Perron (1998).
[8] For more details, the readers are referred to Bai and Perron (2003).

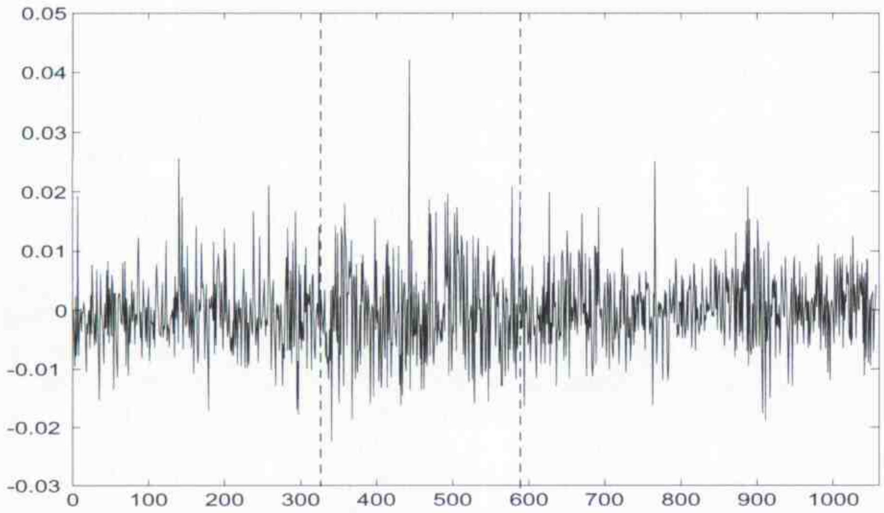

Fig. 1. *Return series of exchange rate euro/US dollar*

first and the last segments ($\hat{h}_1 = \hat{h}_3 = 0.000007$). On the other hand, the estimated value on the second segment is $\hat{h}_2 = 2 \times \hat{h}_1 = 2 \times \hat{h}_3 = 0.000014$.

It seems that the obtained results confirm the recent works of some authors as Mikosch and Starica (2004) who showed the difficulty of describing long financial series with tools retaining the stationarity hypothesis of the covariance structure (long memory for example). In other words, a systematic adaptation of the parameters of the models used to describe long financial series must be done. These results also confirm those obtained by Loretan and Phillips (1994).

VI. CONCLUSION

This article has discussed the problem of selecting the number of breaks and their locations in the covariance structure of a series by adopting a non-parametric approach based on the evolutionary spectral density. The simulation results are globally adequate and indicate that the information criteria having heavy penalty are more precise in the detection of the number of changes. The empirical results relating to the return series of exchange rate euro/US dollar are satisfactory and show that the covariance structure of the series considerably varies and the unconditional volatility appears no constant. Another feature is that these results confirm some recent works existing in the literature.

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